ANALYSIS OF COMMUNICATION PROTOCOLS USING FINITE POPULATION QUEUING SYSTEMS PETRI NETS

Čapek, J.; Dub, M. & Hanzálek, Z
Department of Control Engineering, ČVUT - FEL, Karlovo nám. 13, Prague 2.
{capekj@vision.felk.cvut.cz, hanzalek@rtime.felk.cvut.cz}

Abstract

In the paper we provide an example of the modelling of collisions arising on the communication channel under non-deterministic MAC methods. We present a simple Petri nets abbreviation suitable for modelling of some classes of finite population queuing systems. We present a stochastic Petri net model of a communication channel as well as examples of pure and slotted Aloha viewed as M/D/1 and M/D/1/∝/k queuing systems. Simulation results are demonstrated.

1 Introduction

Simulation is sometimes the only method how to detect some properties of the communication protocols. In the non-deterministic media access methods, perhaps all properties are affected by the collisions on channel. In the paper, first we recall several performance measures presented in literature. Next we introduce the a simple Petri net abbreviation, which is suitable for modelling and simulation of finite population queuing systems. A three state model of a communication channel, on which we demonstrate the collisions, is presented in the third part. The last part shows the throughput analysis of the Aloha models presented in the fourth section.

2 Performance Measures of Communication Protocols

The key to define a general performance measure is to view a network or network device as a general open queuing system as shown in Fig 1. Depending on the specific network model or device operation, the queuing systems have the arrival, departure and recycle processes, queue size, and a service discipline (MAC protocol). For this representation, we identify two parameters that affect the system performance:

(1) Input load (*I*), which is an average number of new jobs arriving to the system from outside of the system boundary over a time interval [1].

(2) Offered load (*G*) (See Fig 1) is an average number of jobs offered to the queuing system over a time interval. When the packet transmission time is of the constant length T, the time interval is T [2]. We suppose that the traffic offered to channel *G* consists not only of new packets but also of previously collided packets (Kleinrock [2]).

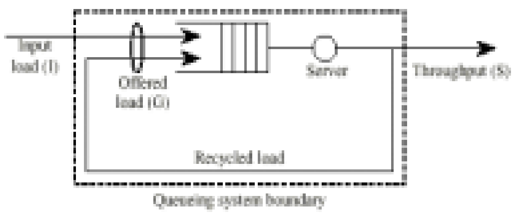

Fig 1: A general queuing model for a network or network device

The two following performance measures are often presented in literature:

1) The Throughput (*S*) is defined as the fraction of nominal network bandwidth that is used for carrying of the successfully transmitted data. Kleinrock, who assumes constant

Acknowledgments
This work was supported by the Ministry of Education of the Czech Republic under Project J04/98:212300013 and by the Project IST –1999-10 504 GCAP

packet length in [2], defines the throughput as an average number of successfully transmitted packets per packet transmission time.

2) The Delay (*D*), often called the transport time, is the average time interval from the moment a new job enters the system boundary until the moment it leaves the boundary.

The *queuing time* or *waiting time* is defined as the average time that a job spends in the queue. For an Ethernet segment for example, the delay is the time that an arriving packet should wait before finding a chance for transmission. In pure Aloha, the waiting time is always zero. For a collision-free network, the service time is the sum of the packet transmission time T and the propagation delay.

3 Finite Population Queuing Systems Petri Nets (FPQSPN)

FPQSPN are high level Petri nets, suitable for modelling of systems, where a number of identical functional elements (the population of k users) share certain resources. The FPQSPN is a facility, which has a power to simulate some types of the queuing systems but remains nearly as graphical as a low level Petri net. From the time behaviour point of view, the FPQSPN correspond to the generalised stochastic Petri nets presented in [5].

A FPSQPN diagram contains a sub-diagram of the population and a sub-diagram of resources. During the unfolding, the population is replaced by a set of *k* identical sub-diagrams. Formally, a FPQSPN is a 10-tuple (**P**, **T**, Pre, Post, M_0, t,t2, tt, s, k) such that:

- **P** *is a finite and non-empty set of m places,*
- **T** *is a finite and non-empty set of n transitions,*
- Pre *is an input function, representing weighted arcs connecting places to transitions called precondition matrix of size* [|**P**|,|**T**|],
- Post *is an output function, called post-condition matrix of size* [|**P**|,|**T**|],
- M_0: $P \Rightarrow \{1, 2, 3, \ldots\}$ *is an initial marking*,
- t: $T \Rightarrow t \in R_0^+$ *is mean time associated to transition*,
- t2: $T \Rightarrow t \in R_0^+$ *second time parameter (variance)*,
- tt: *T is the type of timed transition*
- so: $T, P \Rightarrow \{0, 1\}$ *determines, whether the place or transition is shared object,*
- k: $\in N$ *is number of consumers(population size)*

Places and transitions are either shared objects, graphically represented by double lines, or non-shared objects. Non-shared object is unfolded to the set of *k* same objects. Shared objects are left without change. Unfolding algorithm threats all objects in folded model according to following roles:

1. For a non-shared i^{th} place or non-shared i^{th} transition, create *k* places indexed by $i + m*j$ ($j = [0...n-1]$) or n transitions indexed by $i + n*j$ ($j = [0...n-1]$).
2. A shared i^{th} place or non-shared i^{th} transition, just copy.
3. For an arc between two non-shared objects create *k* arcs and their sources and destinations index analogously to step 1.
4. For an arc between one non-shared object and one shared object, create *k* arcs connected to the same object on the shared object side and non-shared object side index according to point 1.
5. An arc between two shared objects, just copy.

4 FPQSPN Model of Channel

In the Aloha channel, a packet is successfully transmitted, when the actual value of random variable representing inter-arrival time between consecutive packets is at most the length of 'vulnerable interval'[2], which equals to the packet transmission time (assumed, the *propagation delay* (*a*) among all nodes is constant and that a node never starts the transmission during receiving a packet). If other packet is offered to the channel during the interval, a collision occurs. If the channel is at time t in the state of collision and a packet is offered to the media at time *t*, the collision persists until the time *t*+1.

The transition T_1 in Fig 2 is a stochastic time transition with exponential distribution. The Poisson process, defining the token arrivals to place P_1, represents the channel packet arrivals with mean arrival rate $\lambda = 1/t1$ (proof can be found e.g. in [7] pp. 207). Assuming, time t_5, t_6 =1, the mean time, associated to transition T_1, represents reciprocal of offered traffic *G* (in terms of queuing theory it represents the mean arrival rate λ [7]). When we offer a packet to idle channel, the transition T_2 changes the state of channel from idle (a token in P_2) to busy (a token in P_3). A token resides in the place P_3 until the deterministic time transition T_5 is fired or until another packet arrives. Time t_5 equals to the packet transmission time. A firing of the transition T_6 represents a collision resolution. If no other packet arrives until T_6 is fired, the channel returns to the idle state. If a packet arrives during the delay t_6, the transition T_4 is fired. At this moment, transition T_6 is temporarily disabled and the firing of auxiliary transition T_7 initialises the transition T_6 (time t_6 is re-assigned).

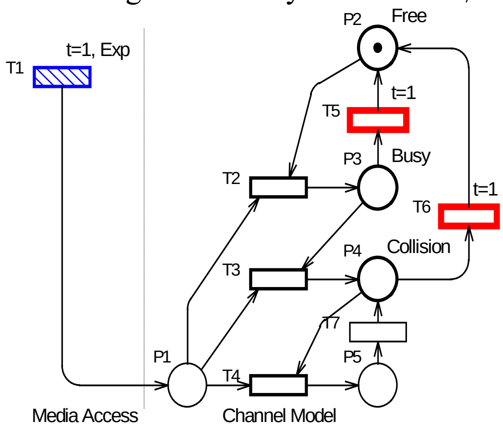

Fig 2: FPQSPN Model of Aloha considered to be M/D/1 queuing system

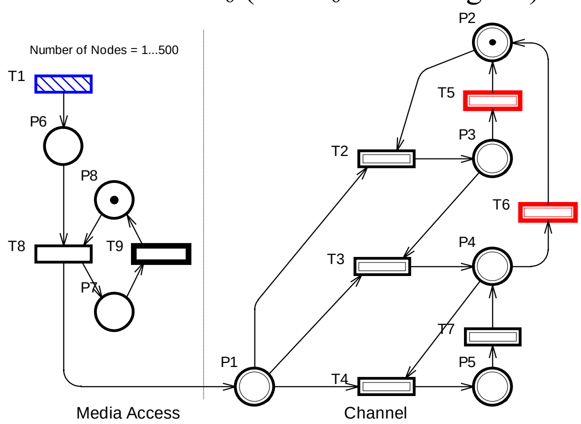

Fig 3: FPQSPN Model of Aloha considered as finite population M/D/1 queuing system

5 FPQSPN Modelling of Aloha Methods Under M/D/1/s/k model

The left part of the FPQSPN model of Aloha network viewed as $M/D/1/\infty/k$ system (See Fig 3) represents *k* nodes (respectively single node, which is during unfolding replaced by a set of *k* nodes).

Arrival process generated by whole population is a sum of Poisson processes defining arrivals to each of *k* nodes. The mean value of offered traffic is given by the sum of the reciprocals of mean times of transitions T_1 in all nodes.

The transitions T_8, T_9 and places P_7, P_8 ensure, that a node does not transmit during its own transmission. The transition T_9 is a deterministic time transition. In order to avoid self-collision, the time t_9 is little longer than the packet transmission time. The Self-collision would occur if the transitions T_9, and T_8 and one of the transitions $\{T_2, T_3, T_4\}$ were been fired before the firing of transition T_5 or T_6. The time of the transition T_9 equals 1.000001.

The FPQSPN model of slotted Aloha considered to be $M/D/1/\infty/k$ system is in Fig 4. The place P_6 represents the immediate transmission buffer. The transitions T_8, T_{12}, T_{11}, T_{10} and the places P_6, P_{10}, P_{11} represent the 'slotting device', which synchronizes the transmissions of all nodes to the beginnings of time slots. The transition T_{11} is a deterministic time transition ($t_{11} = 0.00001$). The transition T_{10} is a deterministic time transition with $t_{10} = 1$. The times $t_{10} + t_{11}$ form a time slot of length 1.00001, which equals to the packet transmission time plus a little delay avoiding a collision. The transition T_{11} is be non-zero time transition in order to ensure that tokens in places corresponding to the place P_6 in all nodes are consumed before firing of T_{11}. The transition T_9, places P_7 and P_8 ensure that a node sends at most one packet per slot time.

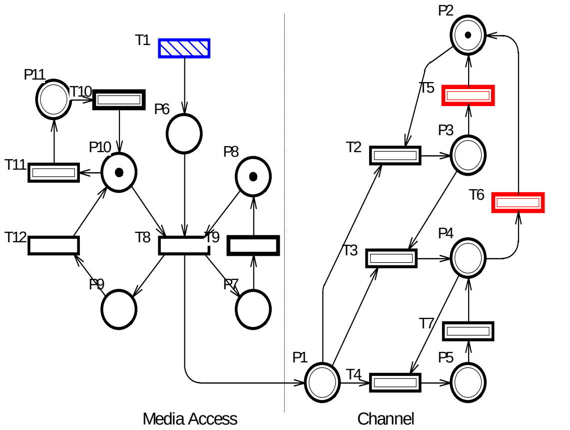

Fig 4: FPQSPN Model of Slotted Aloha considered as finite population M/D/1 queuing system

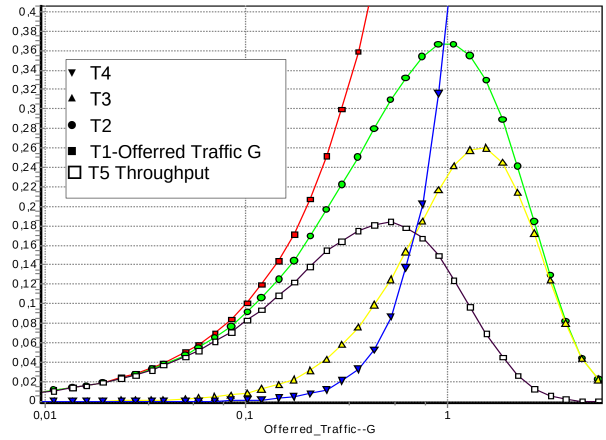

Fig 5: Simulation of model in Fig. 2: The transitions firing frequencies as functions of G in log. coordinates (*G* versus *G* curve is presented for educative purposes)

6 Simulation results

All curves plotted in the figures are functions of the offered traffic *G*. Simulator (StpnPlay [6]) splits the interval of *G* by a geometric progression into *m* steps (in Fig 5 $m = 37$). In each step, it assigns time of transition T_1 according to desired value of *G*, analyses the model in the token player and it computes the firing count vector [4] (*characteristic vector* as defined in [3]). The firing count vector divided by the maximal simulation time is called the *firing frequency vector*. Finally, *m* firing frequency vectors were assigned into the *firing frequencies matrix*. The columns of firing frequencies matrix are plotted in Fig 5 – Fig 7.

In Fig 5, we present the firing frequencies of transitions in Fig 2 as functions of offered traffic $G = \langle 0.01, 5)$. The firing frequency of T_2 means the number of packets, finding the channel free at the beginning of the transmission. The firing frequency of T_3 represents the number of packets causing a collision. The firing frequency of T_4 represents the number of packets finding the channel in collision state. The collision ratio defined as a number of collided packets versus *G* increases when we increase of *G*. The maximal channel throughput equals to 0.18 at $G = 0.5$. In average, for maximal *S*, finds just arrived packet the channel busy with probability 0.12 (T_3 fires) or in collision state (T_4 fires) with probability 0.08. For values $G > 10$, almost all arrived packets participate in collisions. The firing frequency of transition T_3, equals to frequency of collision resolutions. The firing frequency of T_3+T_4 versus T_1 represents the collision ratio.

The throughput curves of pure Aloha network as function of *G* are plotted in Fig.6. The throughput curve of network consisting of 500 nodes is identical to throughput curve under infinite population. For that reason, both characteristics in Fig 6 and Fig 7 are represented by common line. For large *G*, the throughput of Aloha loaded by a small number of nodes is lower than the throughput of "large network". It is because the probability of collision is large for small number of nodes with compare to the probability of collision in traffic generated by large number of nodes. The maximal offered traffic generated by *k* nodes equals to *k* because the offered traffic generated by a node equals at most to 1. For example, when the value of the offered traffic generated by two Aloha nodes approaches two, the structure of traffic is similar to structure of the traffic generated by two deterministic processes each other shifted by a delay. Probability of arising of a collision would approach to one under such conditions.

The throughput of the slotted Aloha network, viewed as a queuing system M/D/1/∝/k, as a function *G* for various number of nodes is plotted in Fig 7. The same explanation as for pure Aloha applies for large *G* and small number of nodes, except that the two deterministic processes are synchronised by “slotted inherence” of the protocol.

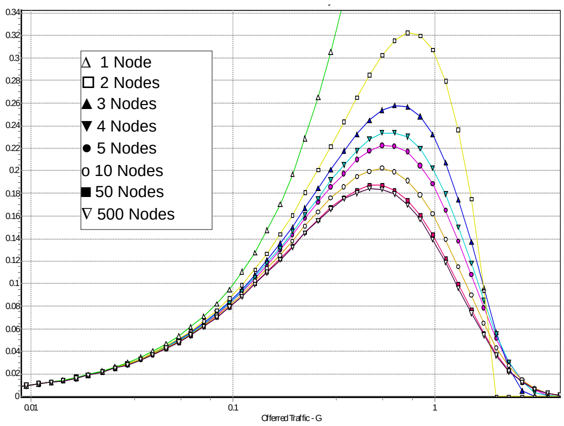

Fig 6: Aloha: The channel throughput *S* as a function of the offered traffic *G*; various numbers of nodes.

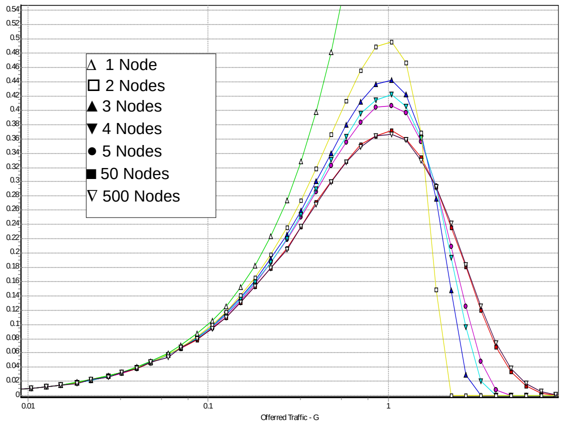

Fig 7: Slotted Aloha M/D/1 finite population: The channel throughput S as a function of the offered traffic *G*; various numbers of nodes

7 Conclusion

In this paper, we modelled the process of arising of collisions on communication channel in the Petri nets. We presented a simple abbreviation of Petri nets, the Finite Population Queuing Systems Petri nets (FPWSPN), which simplifies the analysis of some non-deterministic communication protocols. As an example, we presented the throughput analysis of pure and slotted Aloha MAC protocols using FPQSPN. Although there are many interesting communication protocols used in industry both pure and slotted Aloha seem to be good educational examples of modelling and analyses of non-deterministic communication protocols in FPQSPN.

Authors actually prepare for publication an analysis of the CSMA methods an analysis of the Ethernet. Their actual research is oriented to industry automation communication protocols [8].

8 References

[1] W. Stallings, “Local and Metropolitan Area Networks”, 6/e, Prentice Hall 2000, ISBN: 0-13-012939-9

[2] Kleinrock, L. and Tobagi, F. A., "Packet Switching in radio Channels: Part I—Carrier Sense Multiple Access Modes and Their Throughput Delay Characteristic", IEEE Transactions on Communications., Corn-23, Dec. 1975, pp. 1400-1416.

[3] René D., Hassane A., “Petri Nets and Grafcet” Prentice Hall International (UK) Ltd, 1992, ISBN: 0-13-327537-X

[4] Tadao Murata: “Petri Nets: Properties, Analysis and Applications” Proceedings of the IEEE, vol. 77, No. 4, April 1989.

[5] M. A. Marsan, G. Balbo, and G. Conte "A class of Generalizes Stochastic Petri Nets for the performance analysis of multiprocessor systems”. ACM Transactions on Computer Systems, 2(1):1993-122, May 1984.

[6] Čapek, J. Hanzálek Z, “StpnPlay a modeling and simulation Petri net tool” submitted for publ. in proc. of PNPM2001, univ. of Dortmund, Germany 2001.

[7] Allen Arnold O. "Probability, Statistics, and Queuing Theory: With Computer Science Applications” 2/e Academic Press, Inc. 1990, ISBN: 0-12-051051-0

[8] Čapek, J. - Hanzálek, Z.: STPN Model of Physical and MAC Layer of LonWorks. Proc. Control System Design. Berlin: Springer. 2000. p. 335- 342.